ON SAMPFORD'S PROCEDURE OF UNEQUAL PROBABILITY SAMPLING WITHOUT REPLACEMENT

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1. Introduction: When sampling is done with unequal probabilities and without replacement for estimating the population total Y of a characteristic y defined over a population of size N, Horvitz and Thompson (1952) have proposed an unbiased estimator,

$$\hat{Y} = \sum_{1}^{n} \frac{y_i}{\pi_i}, \tag{1}$$

with variance

$$V(\hat{Y}) = \sum^{N} \frac{Y_i^2}{\pi_i} + \sum_{i}^{N} \sum_{j(\neq i)}^{N} \frac{\pi_{ij}}{\pi_i \pi_j} Y_i Y_j - Y^2 \tag{2}$$

where n is the sample size, π_i is the probability of including the i-th unit in the sample and π_{ij} is the probability for the i-th and j-th units to be both in the sample. When information on an auxiliary characteristic x, assuming the value X_i on the i-th unit, is available for all the units where Y_i is approximately proportional to X_i, considerable reduction in the variance can be achieved by making $\pi_i \alpha X_i$. Such a scheme must obviously satisfy the condition

$$\pi = np_i, \tag{3}$$

where $p_i = X_i/X$, X being the sum of all the X_i's. Condition (3) obviously puts a restriction on the X_i's viz., $\max_i X_i \leq \frac{X}{n}$ which is not a severe one. Among the schemes that satisfy condition (3) and are applicable for general sample size $n \geq 2$, we will consider here the Goodman and Kish (G and K) (1950) procedure and the Sampford's (1967) procedure. The procedure of Sampford, which is a generalization of the Durbin's (1967) procedure for sample size 2, is described as follows:

Assuming without loss of generality that $np_i < 1$ for all i, define

$$\lambda_i = p_i/(1-np_i) \tag{4}$$

Let $L_0 = 1$ (5)

and $L_m = \sum_{S(m)} \lambda_{i_1} \lambda_{i_2} \ldots \lambda_{i_m}, \ (1 \leq m \leq N)$ (6)

where S(m) denotes a set of m different units $i_1, i_2, \ldots, i_m$ and the summation in (6) is over all such possible sets drawn from the population which are $\binom{N}{m}$ in number. The procedure then consists of selecting the particular sample S(n), consisting of units $i_1, i_2, \ldots, i_n$ with probability

$$P\{S(n)\} = nK_n \lambda_{i_1} \lambda_{i_2} \ldots \lambda_{i_n} (1 - \sum_{u=1}^{n} p_{i_u}) \tag{7}$$

where

$$K_n = (\sum_{t=1}^{n} t L_{n-t}/n^t)^{-1} \tag{8}$$

Since the evaluation of the expression in (7) for all the $\binom{N}{m}$ sets is out of question in practice, Sampford has suggested two alternative ways of achieving the probabilities $P\{S(n)\}$ as given in (7). Method (i) is to select the units without replacement, with the probabilities evaluated at each drawing according to the rule described and illustrated by Sampford in his article. Method (ii) is to select n units with replacement, the first drawing being made with probabilities p_i and all subsequent ones with probabilities proportional to $p_i/(1-np_i)$ and rejecting completely any sample that doesn't contain n distinct units and to start afresh. In practice method (ii) could be convenient because a sample can be discarded as soon as a duplicate unit is drawn. However for small samples one may take as a guide line in the relative preference of methods (i) and (ii), the value of the expected number of samples that must be drawn to obtain an acceptable sample which is given by $K_n \cdot (\Sigma\lambda_t)^{n-1}/(n-1)!$. For this scheme of sampling Sampford has shown that π_i is given by (3) and

$$\pi_{ij} = K_n \cdot \lambda_i \lambda_j \phi_{ij} \tag{9}$$

where

$$\phi_{ij} = n \cdot \sum_{\substack{S(n-2) \\ i,j \notin S}} \lambda_{\ell_1} \lambda_{\ell_2} \ldots \lambda_{\ell_{n-2}} \cdot \{1 - (p_i + p_j) - \sum_{u=1}^{n-2} p_{\ell_u}\} \tag{10}$$

It has also been shown by Sampford that the condition $\pi_i \pi_j - \pi_{ij} > 0$, is satisfied which ensures the nonnegativity of the Yates and Grundy variance estimator.

Even though the exact expression for π_{ij} of the Sampford's procedure is available, the computations become quite cumbersome particularly for N and/or n large. Since the simplicity of computations is one of the factors to be considered in choosing a sampling procedure, it will be of advantage if reliable approximate expressions for π_{ij} are derived because one may prefer to use the approximate expressions that would be quite satisfactory and easy for numerical evaluation. Also since the procedures of Goodman and Kish, and Sampford are two competitive schemes, it will be worth while if we could compare the efficiencies of the two schemes. Thus it would be realistic for comparison purposes to derive the approximate expressions for π_{ij} and hence the variance for the Sampford's procedure using the asymptotic approach of Hartley and Rao (1962). In order to evaluate the variance expression of the Horvitz-Thompson (H. T.) estimator under the Sampford's procedure, we will first evaluate π_{ij} correct to $O(N^{-4})$ under the assumptions of Hartley and Rao (1962) viz., n is small relative to N and p_i is of $O(N^{-1})$.

2. Evaluation of the approximate expression for π_{ij} of the Sampford's procedure: Since $np_i < 1$, from (4) we get by expanding in Taylor's series and retaining terms up to $O(N^{-4})$ only,

$$\lambda_i \lambda_j = p_i p_j \{1 + n(p_i + p_j) + n^2(p_i^2 + p_j^2 + p_i p_j)\} \tag{11}$$

Since the leading term in $\lambda_i \lambda_j$ above is of $O(N^{-2})$, in order to evaluate π_{ij} in (9) correct to $O(N^{-4})$ only, it would be sufficient to evaluate K_n and ϕ_{ij} each correct to $O(N^{-2})$. For evaluating K_n and ϕ_{ij} we need the following lemma.

Lemma 1: Let $\ell_1, \ell_2 \ldots \ell_m$ be the units drawn in that order when a simple random sample without replacement of size m is drawn from a population of N units. Let $p_1, p_2 \ldots p_N$ be such that each p_i is of $O(N^{-1})$ and $\sum_1^N p_i$ is not necessarily equal to one. Then for $m \geq 3$ where m is small relative to N, the following relations are true correct to $O(N^{-2})$:

$$N_{(m)} \cdot E(p_{\ell_1} p_{\ell_2} \cdots p_{\ell_m}) = (\Sigma p_t)^m - \binom{m}{2}(\Sigma p_t)^{m-2} \cdot \Sigma p_t^2 + 2 \cdot \binom{m}{3} \cdot (\Sigma p_t)^{m-3} \cdot \Sigma p_t^3 + 3 \cdot \binom{m}{4} \cdot (\Sigma p_t)^{m-4} \cdot (\Sigma p_t^2)^2 \tag{12}$$

$$N_{(m)} \cdot E(p_{\ell_1}^2 p_{\ell_2} p_{\ell_3} \cdots p_{\ell_m}) = (\Sigma p_t)^{m-1} \cdot \Sigma p_t^2 - (m-1)(\Sigma p_t)^{m-2} \cdot \Sigma p_t^3 - \binom{m-1}{2} \cdot (\Sigma p_t)^{m-3} \cdot (\Sigma p_t^2)^2 \tag{13}$$

$$N_{(m)} \cdot E(p_{\ell_1}^3 p_{\ell_2} p_{\ell_3} \cdots p_{\ell_m}) = (\Sigma p_t)^{m-1} \cdot \Sigma p_t^3 \tag{14}$$

and

$$N_{(m)} \cdot E(p_{\ell_1}^2 p_{\ell_2}^2 p_{\ell_3} p_{\ell_4} \cdots p_{\ell_m}) = (\Sigma p_t)^{m-2} \cdot (\Sigma p_t^2)^2 \tag{15}$$

wherein $N_{(m)} = N(N-1)\ldots(N-m+1)$ and $\binom{\mu}{v}$ is to be taken as zero if $\mu < v$.

Proof is by induction which is straightforward and hence is omitted.

Remark: Even though the proof of the lemma assumes that $m \geq 3$, the relations (12)-(15) are true for m = 0, 1, and 2 also which can be easily verified.

L_m of (6) can be written as

$$L_m = \binom{N}{m} \cdot E[\lambda_{\ell_1} \lambda_{\ell_2} \cdots \lambda_{\ell_m}] \tag{16}$$

where E denotes the expectation over the scheme described in Lemma 1.
Substituting the value of λ_{ℓ_i} from (4) and by expanding in Taylor's series we get from (16)

$$L_m = \frac{N_{(m)}}{m!} \cdot E[\prod_{i=1}^{m} p_{\ell_i} \{1 + np_{\ell_i} + n^2 p_{\ell_i}^2 + \ldots\}]. \tag{17}$$

Now, it can be easily seen that for any set of positive integers $\alpha_1, \alpha_2, \ldots, \alpha_m$; the contribution of $N_{(m)} \cdot E[p_{\ell_1}^{\alpha_1} p_{\ell_2}^{\alpha_2} \cdots p_{\ell_m}^{\alpha_m}]$ to L_m, correct to $O(N^{-2})$, would be zero if $\sum_1^m \alpha_i > (m+2)$. Further from the basic properties of simple random sampling it is also known that $E[p_{\ell_1}^{\alpha_1} p_{\ell_2}^{\alpha_2} \cdots p_{\ell_m}^{\alpha_m}]$ is the same for all the permutations of $(\alpha_1, \alpha_2, \ldots, \alpha_m)$. Hence from (17) it follows that the expression for L_m that could contribute to $O(N^{-2})$ is

$$L_m = \frac{N_{(m)}}{m!}[E(p_{\ell_1} p_{\ell_2} \cdots p_{\ell_m})$$

$$+ nm \cdot E(p_{\ell_1}^2 p_{\ell_2} p_{\ell_3} \cdots p_{\ell_m})$$

$$+ \frac{n^2 m(m-1)}{2} \cdot E(p_{\ell_1}^2 p_{\ell_2}^2 p_{\ell_3} p_{\ell_4} \cdots p_{\ell_m})]$$

Substituting from (12)-(15) we get for $m \geq 3$,

$$L_m = \frac{1}{m!}[1 + \{\binom{m}{1}n - \binom{m}{2}\} \cdot \Sigma p_t^2$$

$$+ \{\binom{m}{1}n^2 - 2 \cdot \binom{m}{2}n + 2\binom{m}{3}\} \cdot \Sigma p_t^3$$

$$+ \{\binom{m}{2}n^2 - 3 \cdot \binom{m}{3}n + 3 \cdot \binom{m}{4}\} \cdot (\Sigma p_t^2)^2] \tag{18}$$

It can be verified that (18) in fact holds for $m = 0, 1,$ and 2 also with the convention that $\binom{\mu}{v} = 0$ if $\mu < v$.

Theorem 1: For $n \geq 5$, the expression for $\frac{1}{K_n}$ correct to $O(N^{-2})$ is

$$\frac{1}{K_n} = \frac{1}{(n-1)!} + \frac{n}{2(n-2)!}\Sigma p_t^2 + \frac{n(n+1)}{3(n-2)!}\Sigma p_t^3$$

$$+ \frac{n(n+1)(n-2)}{8(n-2)!}(\Sigma p_t^2)^2 \tag{19}$$

Proof: From (8) we get by using the transformation $s = n-t$,

$$\frac{1}{K_n} = \sum_{s=0}^{n-1}(n-s) \cdot L_s / n^{n-s}$$

$$= \frac{L_0}{n^{n-1}} + \frac{(n-1) \cdot L_1}{n^{n-1}} + \frac{(n-2) \cdot L_2}{n^{n-2}} + G, \tag{20}$$

where $G = \sum_{s=3}^{n-1}(n-s) \cdot L_s / n^{n-s}$. Let $T_s = n^s / s!$, and for any nonnegative integers $0 \leq \ell \leq m$, let

$$I_{(\ell, m)} = \sum_{s=\ell}^{m} T_s$$

and

$$J_{(\ell, m)} = \sum_{s=\ell}^{m} s \cdot T_s$$

Then by using the relations

$$\sum_{s=\ell}^{m}(n-s) \cdot T_{s-\alpha} = (n-\alpha) \cdot I_{(\ell-\alpha, m-\alpha)} - J_{(\ell-\alpha, m-\alpha)}$$

for any $\alpha \leq \ell \leq m$;

$$J_{(\ell, m)} = n \cdot I_{(\ell-1, m-1)};$$

and

$$I_{(\ell+1, m+1)} - I_{(\ell, m)} = T_{m+1} - T_\ell,$$

we get by substituting the value of L_s from (18) into (20),

$$\frac{1}{K_n} = \frac{1}{(n-1)!} + \frac{n}{2(n-2)!}\Sigma p_t^2 + \frac{n(n+1)}{3(n-2)!}\Sigma p_t^3$$

$$+ \frac{n(n+1)(n-2)}{8(n-2)!}(\Sigma p_t^2)^2 \qquad \text{Q.E.D.}$$

Remark: Direct evaluation of $\frac{1}{K_n}$ from (8) for $n = 2, 3,$ and 4 shows that (19) in fact holds for $n \geq 2$. From (19) we get for $n \geq 2$

$$K_n = \{(n-1)!\}[1 - \frac{n(n-1)}{2}\Sigma p_t^2 - \frac{n(n-1)(n+1)}{3}\Sigma p_t^3$$

$$+ \frac{n(n-1)(n^2-n+2)}{8}(\Sigma p_t^2)^2] \tag{21}$$

correct to $O(N^{-2})$.

As the expression for ϕ_{ij} in (10) is not meaningful to consider for $n=2$, we derive the approximate expression for ϕ_{ij} assuming $n \geq 3$. Expression (10) can alternatively be written as

$$\phi_{ij} = n \cdot \binom{N-2}{n-2} \cdot E'[\lambda_{\ell_1} \lambda_{\ell_2} \cdots \lambda_{\ell_{n-2}}$$

$$\{1 - (p_i + p_j) - \sum_{u=1}^{n-2} p_{\ell_u}\}] \tag{22}$$

where E' denotes the expectation taken over the scheme of selecting (n-2) units from the population excluding the i-th and j-th units with simple random sampling without replacement. Using the results of Lemma 1 with suitable modifications we get, from (22), for $n \geq 3$,

$$\phi_{ij} = \frac{n}{(n-2)!}[1 + \{\frac{(n-2)(n+1)}{2} \cdot \Sigma p_t^2 - (n-1)(p_i + p_j)\}$$
$$+ \{(n-1)(n-2)p_i p_j - (n-2)(p_i^2 + p_j^2)$$
$$- \frac{(n-2)(n^2-3)}{2}(p_i + p_j)\Sigma p_t^2$$
$$+ \frac{(n-2)(n^2+2n+3)}{3}\Sigma p_t^3$$
$$+ \frac{(n-2)(n-3)(n^2+3n+4)}{8} \cdot (\Sigma p_t^2)^2\}], \tag{23}$$

correct to $O(N^{-2})$.

Sampford's procedure is a generalization, for sample size $n \geq 2$, of the Durbin's (1967) procedure which is for sample size 2. The expression for π_{ij} of the Durbin's procedure is

$$\pi_{ij} = K_2 p_i p_j (\frac{1}{1-2p_i} + \frac{1}{1-2p_j})$$

Substituting the value of K_2 from (21) in the above, we get after retaining terms to $O(N^{-4})$ only,

$$\pi_{ij} = 2p_i p_j [1 + \{(p_i + p_j) - \Sigma p_t^2\} + \{2(p_i^2 + p_j^2) - 2\Sigma p_t^3$$
$$- (p_i + p_j)\Sigma p_t^2 + (\Sigma p_t^2)^2\}] \tag{24}$$

Substituting from (11), (21), and (23) into (9), we get for $n \geq 3$,

$$\pi_{ij} = n(n-1)p_i p_j [1 + \{(p_i + p_j) - \Sigma p_t^2\}$$
$$+ \{2(p_i^2 + p_j^2) - 2\Sigma p_t^3 - (n-2)p_i p_j$$
$$+ (n-3)(p_i + p_j)\Sigma p_t^2 - (n-3)(\Sigma p_t^2)^2\}] \tag{25}$$

correct to $O(N^{-4})$.

Observation of (24) shows that (25) is in fact true for $n \geq 2$.

3. Comparison of the variances of the corresponding H. T. estimators for the Sampford's procedure and the Goodman and Kish procedure: The expression for π_{ij} of the Goodman and Kish procedure correct to $O(N^{-4})$, as derived by Hartley and Rao (1962)(5.15 of p. 369), can be written in the modified form as

$$\pi_{ij} = n(n-1)p_i p_j [1 + \{(p_i + p_j) - \Sigma p_t^2\}$$
$$+ \{2(p_i^2 + p_j^2) - 2\Sigma p_t^3 + 2p_i p_j - 3(p_i + p_j)$$
$$\cdot \Sigma p_t^2 + 3(\Sigma p_t^2)^2\}] \tag{26}$$

Theorem 2: Given any unequal probability sampling scheme for selecting a sample of size n whose π_i and π_{ij} are given by

$$\pi_i = n p_i \tag{27}$$

and
$$\pi_{ij} = n(n-1)p_i p_j [1 + \{(p_i + p_j) - \Sigma p_t^2\}$$
$$+ \{2(p_i^2 + p_j^2) - 2\Sigma p_t^3 + a_n \cdot p_i p_j$$
$$- (a_n + 1)(p_i + p_j)\Sigma p_t^2$$
$$+ (a_n + 1)(\Sigma p_t^2)^2\}], \tag{28}$$

correct to $O(N^{-4})$, where a_n is some constant that does not depend on p_t's but may depend on n, the variance expression correct to $O(N^0)$ of the corresponding H. T. estimator is given by

$$V(\hat{Y}_{H.T.}) = \frac{1}{n}[\Sigma p_i z_i^2 - (n-1)\Sigma p_i^2 z_i^2]$$
$$- \frac{(n-1)}{n} \cdot [2\Sigma p_i^3 z_i^2 - \Sigma p_i^2 \cdot \Sigma p_i^2 z_i^2$$
$$- a_n \cdot (\Sigma p_i^2 z_i)^2], \tag{29}$$

$$\text{where } z_i = \frac{Y_i}{p_i} - Y. \tag{30}$$

Proof: Substituting the values of π_i and π_{ij} from (27) and (28) in

$$V(\hat{Y}_{H.T.}) = \Sigma \frac{Y_i^2}{\pi_i} + \sum_i \sum_{j(\neq i)} \frac{\pi_{ij}}{\pi_i \pi_j} Y_i Y_j - Y^2,$$

simplifying and retaining terms to $O(N^0)$ we get the expression in (29).

Q. E. D.

From (25) and (26) it can be observed that condition (28) of Theorem 2 is satisfied for both the procedures of Sampford as well as Goodman and Kish, the values of a_n being $-(n-2)$ and 2 respectively. Since condition (27) is known to be satisfied for both the procedures, we get from (29) that:
Variance of the H. T. estimator for the Sampford's procedure correct to $O(N^0)$ is

$$V(\hat{Y}_{H.T.})_{Samp} = \frac{1}{n}[\Sigma p_i z_i^2 - (n-1)\Sigma p_i^2 z_i^2]$$

$$-\frac{(n-1)}{n}\cdot[2\Sigma p_i^3 z_i^2 - \Sigma p_i^2 \cdot \Sigma p_i^2 z_i^2$$

$$+ (n-2)\cdot(\Sigma p_i^2 z_i)^2\,] \qquad (31)$$

and the variance of the H. T. estimator for the Goodman and Kish procedure correct to $O(N^0)$ is

$$V(\hat{Y}_{H.T.})_{G\text{ and }K} = \frac{1}{n}[\Sigma p_i z_i^2 - (n-1)\Sigma p_i^2 z_i^2]$$

$$-\frac{(n-1)}{n}\cdot[2\Sigma p_i^3 z_i^2 - \Sigma p_i^2 \cdot \Sigma p_i^2 z_i^2$$

$$- 2(\Sigma p_i^2 z_i)^2\,] \qquad (32)$$

From (31) and (32) it follows that when the variance is considered to $O(N^1)$ only,

$$V(\hat{Y}_{H.T.})_{Samp} = V(\hat{Y}_{H.T.})_{G\text{ and }K}$$

$$= \frac{1}{n}[\Sigma p_i z_i^2 - (n-1)\Sigma p_i^2 z_i^2]; \qquad (33)$$

and when the variance is considered to $O(N^0)$,

$$V(\hat{Y}_{H.T.})_{G\text{ and }K} - V(\hat{Y}_{H.T.})_{Samp}$$

$$= (n-1)\cdot(\Sigma p_i^2 z_i)^2 \geq 0 \qquad (34)$$

Further percentage gain in efficiency of the Sampford's procedure over the Goodman and Kish procedure is

$$E = \frac{(n-1)\cdot(\Sigma p_i^2 z_i)^2}{V(\hat{Y}_{H.T.})_{G\text{ and }K}} \times 100 \qquad (35)$$

E will be an increasing function of the sample size since the numerator increases and the denominator decreases as the sample size increases. Thus from (33), (34) and (35) it can be concluded that

Theorem 3: when the variance is considered to $O(N^1)$, the H. T. estimators corresponding to the Sampford's procedure and the Goodman and Kish procedure are equally efficient; and when the variance is considered to $O(N^0)$, H. T. estimator corresponding to the Sampford's procedure is always more efficient than the H. T. estimator corresponding to the Goodman and Kish procedure and the percentage gain in efficiency will be larger for larger sample sizes.

Thus, this result is a generalization of the result due to Rao (1965) wherein he compared the H. T. estimators corresponding to the Durbin's procedure and the Goodman and Kish procedure for sample size 2.

4. Numerical Illustration: The data relates to that of 35 Scottish farms, considered by Sampford (1962). In order to have an idea as to how good the approximate expressions for π_{ij} are in a given situation, the π_{ij} are calculated for the above data by using both the exact as well as the approximate expressions for samples of size 3. The variance also is evaluated using both the sets of π_{ij}. The set of probabilities $\pi_{1j}(j = 2, 3, \ldots, 35)$ are shown along with the approximate $\pi_{1j}(j = 2, 3, \ldots, 35)$ in Table 1. Variance calculated using the exact π_{ij} is $V(\hat{Y}) = 68319$, whereas the variance calculated using the approximate π_{ij} is $V(\hat{Y}) = 68341$, which suggests that in many practical situations the approximate expressions for π_{ij} given in (25) will serve the purpose quite adequately. In Table 2 are presented the variances computed to various orders for both the procedures of Sampford as well as Goodman and Kish when samples of size 4 are considered. The value computed to $O(N^2)$ represents the true variance of the customary estimator in the varying probability with replacement procedure. Values of the successive approximations suggest that the convergence is quite satisfactory even though the population size, $N = 35$, is much smaller than the sizes usually encountered in practice. For larger sample sizes, the relative difference between the two variances correct to $O(N^0)$ is however expected to be much higher than it is in this case.

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Table 1.
Exact π_{1j}'s and the approximate π_{1j}'s of the Sampford's procedure with n = 3 .

Unit No. j	Exact π_{1j}	Approximate π_{1j}	Unit No. j	Exact π_{1j}	Approximate π_{1j}
2	.000439	.000439	19	.001249	.001250
3	.000456	.000457	20	.001249	.001250
4	.000510	.000510	21	.001396	.001397
5	.000527	.000528	22	.001396	.001397
6	.000527	.000528	23	.001712	.001713
7	.000545	.000546	24	.001787	.001788
8	.000572	.000572	25	.001891	.001891
9	.000572	.000572	26	.002185	.002185
10	.000599	.000599	27	.002512	.002512
11	.000625	.000626	28	.002765	.002765
12	.000652	.000653	29	.002794	.002794
13	.000688	.000688	30	.002873	.002873
14	.000796	.000796	31	.003001	.003001
15	.000804	.000805	32	.003061	.003060
16	.000813	.000814	33	.003321	.003319
17	.000850	.000850	34	.003870	.003866
18	.000976	.000977	35	.004077	.004072

Table 2.
Approximations to $V(\hat{Y}_{H.T.})$

Order of approximation	Sampford's procedure	Goodman and Kish procedure
$O(N^2)$	55852	55852
$O(N^1)$	49321	49321
$O(N^0)$	48952	48979